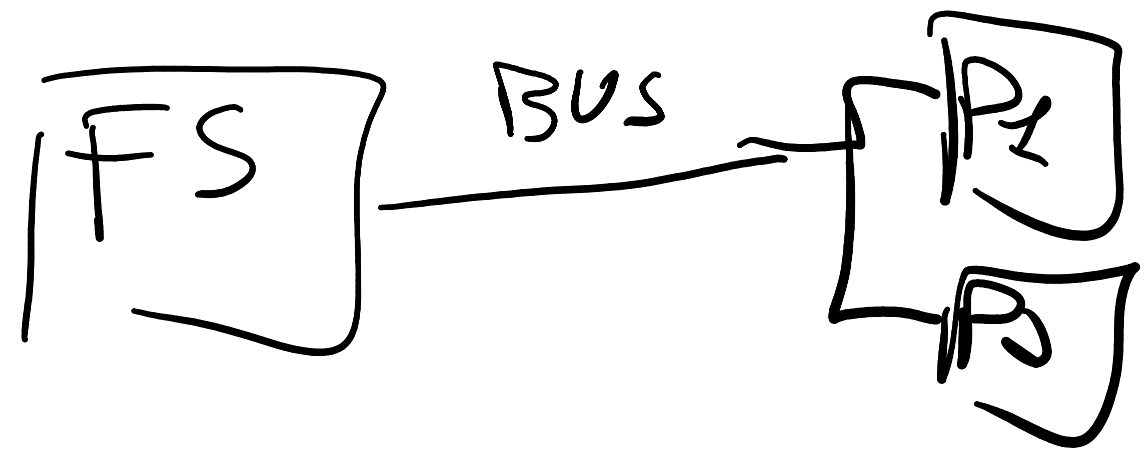
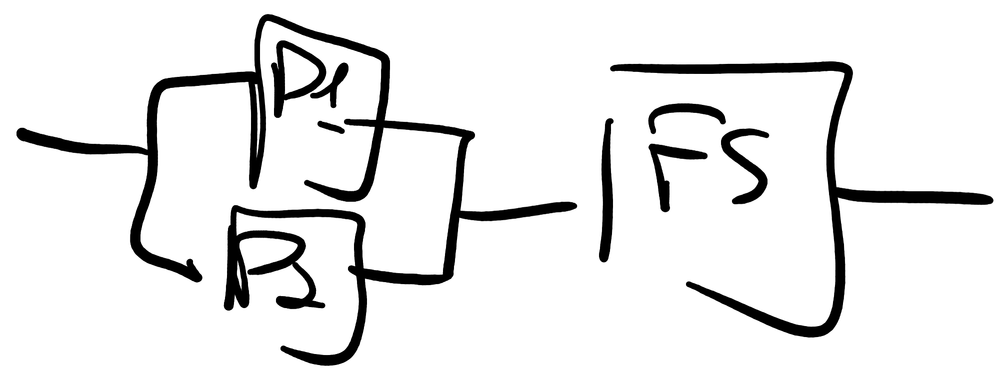
Exercice 1



a)



b)

$$\xi(x) = \begin{cases} 0 & \text{si le système ne fonctionne pas avec l'état } \underline{x} \\ 1 & \text{si le système fonctionne avec l'état } \underline{x} \end{cases}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} P_1 \text{ marche?} \\ P_2 \text{ marche?} \\ P_3 \text{ marche?} \end{pmatrix}$$

$$\xi(x) = x_3 \cdot \left(1 - (1 - x_1)(1 - x_2)\right)$$

c) Importance structurelle d'un composant:

Definition

The **structural importance** of a component c_i in a coherent system of n components is

$$I_\xi(i) = \frac{1}{2^{n-1}} \sum [\xi(1_i, \mathbf{x}) - \xi(0_i, \mathbf{x})]$$

$n = 3$

$$I_\xi(3) = \frac{1}{4} \sum \left(\xi(1_i, \underline{x}) - \xi(0_i, \underline{x})\right) \quad (FS)$$

$$= \frac{1}{4}(1+1+1+0-0-0-0-0) = \frac{3}{4}$$

$$I_\xi(2) = \frac{1}{2^{n-1}} \sum \xi(1_i, \underline{x}) - \xi(0_i, \underline{x})$$

$$= \frac{1}{4}\left(0 + 0 + \cancel{1}1 - 0 - 0 - \underline{1} - 0\right)$$

$$= \frac{1}{4}$$

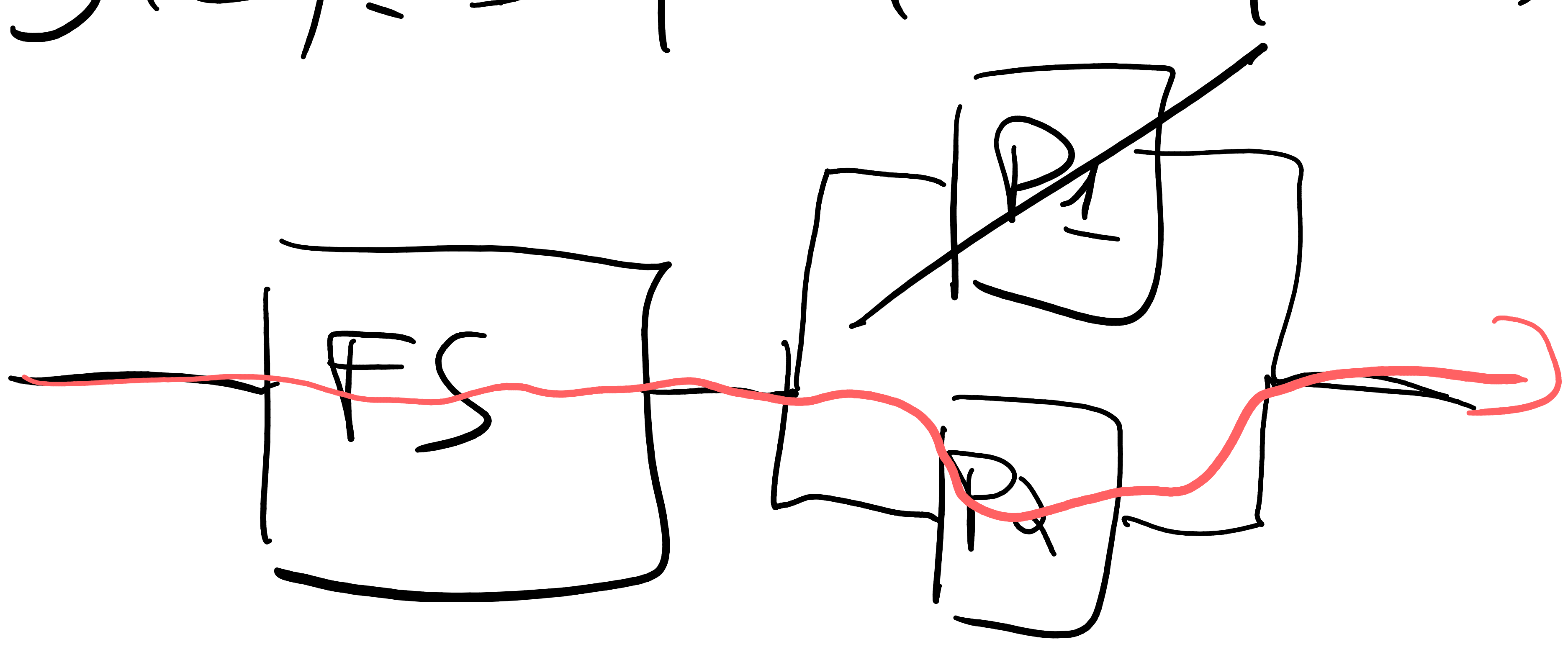
$$I_\xi(1) = \frac{1}{4}\left(0 + 0 + 1 + 1 - 0 - 0 - 1 - 0\right) = \frac{1}{4}.$$

d) minimal path vector : $\underline{x}_{min} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

ou $\underline{x}_{min} = \begin{pmatrix} \cancel{1} \\ 0 \\ 1 \end{pmatrix}$

e) Avec le vecteur de chemin minimal :

$$\xi(\underline{x}) = 1 \cdot \left(1 - (1-0)(1-1)\right) = 1.$$



Definition

A **path vector** for a coherent system is a vector $\mathbf{x}$ such as $\xi(\mathbf{x}) = 1$.

Definition

A **minimal path** for a coherent system is a path vector $\mathbf{x}$ such as $\xi(\mathbf{y}) = 0$ for all $\mathbf{y} < \mathbf{x}$.

f) minimal cut vector:

Definition

A **cut vector** for a coherent system is a vector $\mathbf{x}$ such as $\xi(\mathbf{x}) = 0$.

Definition

A **minimal cut vector** for a coherent system is a cut vector $\mathbf{x}$ such as $\xi(\mathbf{y}) = 1$ for all $\mathbf{y} > \mathbf{x}$.

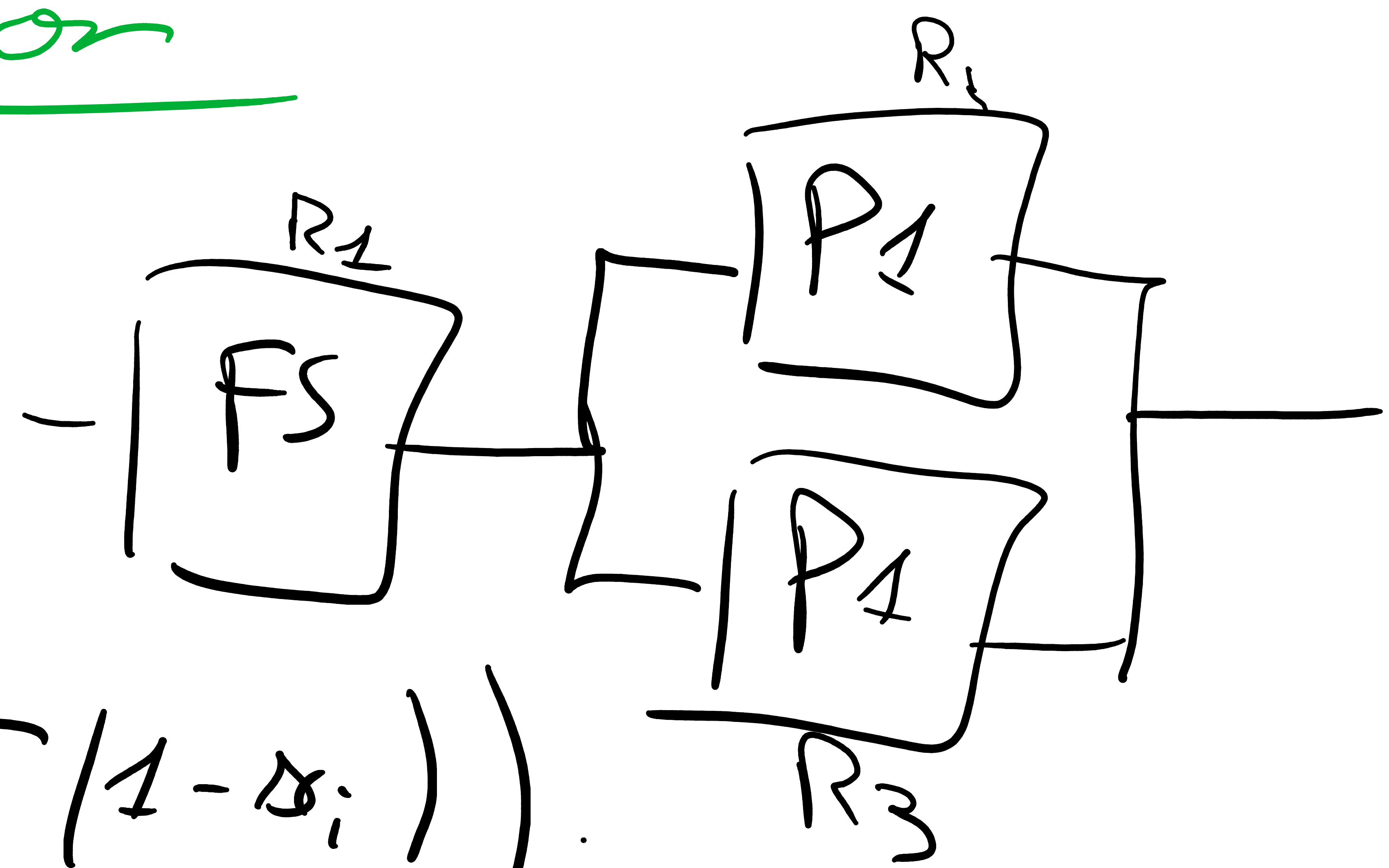
Definition

A **minimal cut set** C_j for a coherent system is a set with all components associated to a given minimal cut vector.

Correction

Exercice 1 :

a) State diagram :



b) $\xi(x) = x_1\left(1-\prod_{i=2}^{n}(1-x_i)\right)$.

c)

x_1	x_2	x_3	$\xi(x)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$I_\xi(x_1) = \frac{1}{2^{3-1}}(3-0) = \frac{3}{4}$

$I_\xi(x_2) = I_\xi(x_3) = \frac{1}{4}(2-1)$

Definition
A **path vector** for a coherent system is a vector $\mathbf{x}$ such as $\xi(\mathbf{x}) = 1$.

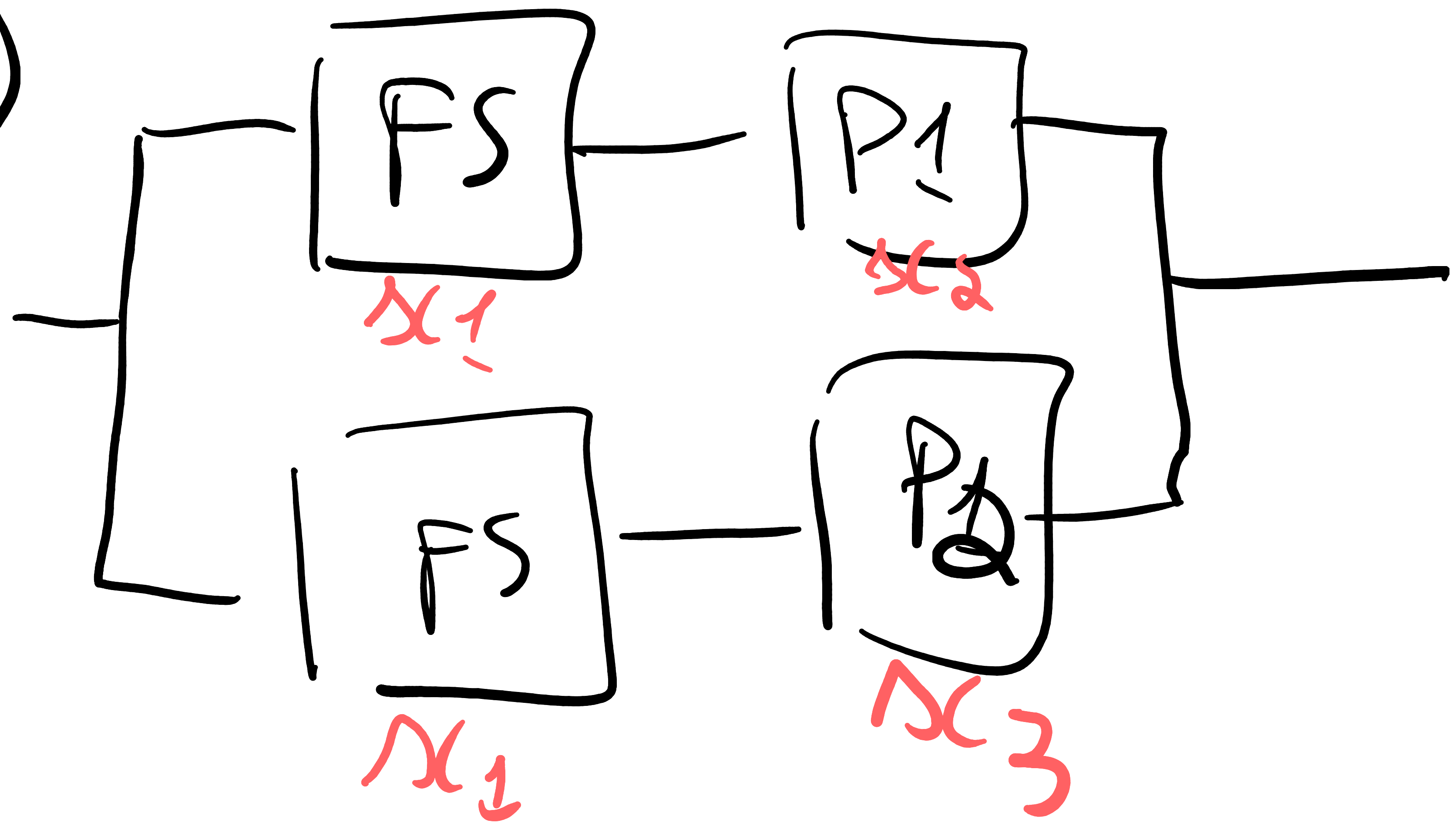
Definition
A **minimal path** for a coherent system is a path vector $\mathbf{x}$ such as $\xi(\mathbf{y}) = 0$ for all $\mathbf{y} < \mathbf{x}$.

d) Minimal path vector

le "+petit vecteur" (i.e celui avec le + de zéros) qui conduit au fonctionnement du système :

$\vec{V}_{min} = 101$ ou $\vec{V}_{min} = 110$.

e)



f) Minimal cut vectors

$$\boxed{\begin{aligned}\vec{C_1} &= 011\\ \vec{C_2} &= 100\end{aligned}}$$

↳ i.e + "grands" vecteurs conduisant à une erreur (ceux avec le + de 1).

Definition

A **cut vector** for a coherent system is a vector $\mathbf{x}$ such as $\xi(\mathbf{x}) = 0$.

Definition

A **minimal cut vector** for a coherent system is a cut vector $\mathbf{x}$ such as $\xi(\mathbf{y}) = 1$ for all $\mathbf{y} > \mathbf{x}$.

Definition

A **minimal cut set** C_j for a coherent system is a set with all components associated to a given minimal cut vector.

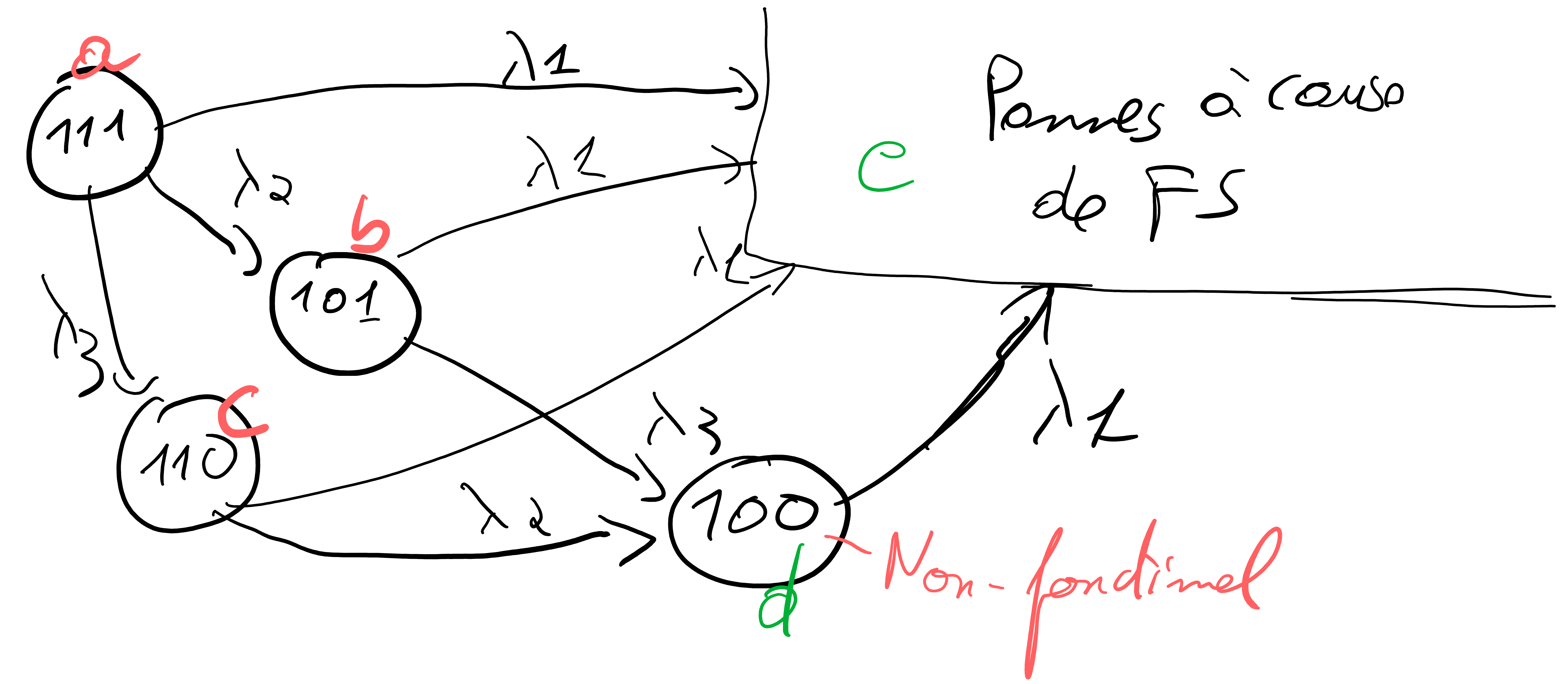
g) $\xi(\underline{x}) = x_1(1-(1-x_2)(1-x_3))$

$= 1-(1-x_1x_3)(1-x_1x_3)$.

h) FS → panne avec proba λ_1

P_1 → panne avec proba λ_2

P_2 → panne avec proba λ_3



i) M : matrice des transitions : $\left[\frac{dP}{dt}\right] = [M] \cdot [P]$

$$
M = \begin{array}{c|ccccc}
\text{De} \backslash \text{Vers} & a & b & c & d & e \\
\hline
a & -(\lambda_1+\lambda_2+\lambda_3) & \lambda_2 & \lambda_3 & 0 & \lambda_1 \\
b & 0 & -(\lambda_1+\lambda_3) & 0 & \lambda_2 & \lambda_1 \\
c & 0 & 0 & -(\lambda_1+\lambda_2) & \lambda_3 & \lambda_1 \\
d & 0 & 0 & 0 & -\lambda_1 & \lambda_1 \\
e & 0 & 0 & 0 & 0 & 0
\end{array}
\qquad
\begin{pmatrix} P_a \\ P_b \\ P_c \\ \vdots \\ \vdots \end{pmatrix}
$$

$P_i(t+\Delta t) = P_i(t)\left[1 - \sum_{i \neq j} s_{i,j}\right]$

$\sum_{\substack{i \neq j \\ j}} P_j(t) \cdot s_{j,i}$

Sur chaque ligne, la somme des éléments vaut 0.