



Side-Channel Analysis

An introduction

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Contents

Introduction

Power Consumption and EM Radiation Introduction Simple Power Analysis

Differential Power Analysis

Basic Principles The Leakage Model Analysis Algorithm

Further Attacks

Generalizations of DPA Computation Time

Conclusion





- Understanding the notion of side-channel analysis (SCA)
- Understanding classic side-channel attacks
- Understanding counter-measures against side-channel attacks



General Context

Algorithm

Implementation

- Hardware (ASIC, FPGA...)
- Software running on a processor (soft-core on an FPGA, micro-controller in an embedded system, general purpose CPU, specialized processor)
- With a specific security objective
 - · Confidentiality (example: cipher algorithm)
 - · Authentification (example: PIN code verification)

• . . .

Handling a secret (can be the algorithm itself) that must not be accessible to the adversary





- Example: Cryptographic algorithm implemented on a smart card
- Input: plain text message
- Output: encrypted message
- By construction, the cryptographic key, which is embedded within the smart card, is not accessible via any operation on the input/output interface of the card.







- KERCKHOFFS principle: P, C et E are public, security depends on K, which is unknown to the adversary
- There are numerous robust algorithms following this model



Cryptanalysis vs Reality...



[Source: https://www.xkcd.com/538/]







Additional input/output channels: Side-channels

- Electromagnetic radiation (EM)
- Power consumption
- Computation time
- . . .



Side-channel Attacks

- Side-channels depend on the implementation of an algorithm:
 - In software
 - In hardware
- Side-channels cannot be observed on the algorithmic (mathematical, cryptanalytic) level.
- The implementation may leak sensitive information (secrets) via side-channels, even if those secrets never appear on the input/output interface.
- As a consequence, a passive observation can allow an attacker to get hold of the secret!



```
Concrete Example
Function verifying a PIN code
```

```
boolean verifyPIN(byte[] inputPIN)
{
  for (int i = 0; i < correctPIN.length; i++)
    if (inputPIN[i] != correctPIN[i])
      return false;
  return true;
}</pre>
```

- Suppose that the arrays inputPIN and correctPIN have size 4 and contain digits only (0–9)
 - What is the complexity of an exhaustive search (try all the PINs)?
 - Can the attacker be smarter than that?





- The attacker can measure the function's execution time
- Note that the function returns once it finds a wrong digit
- The attacker can try 0xxx, 1xxx, ..., 9xxx
- One of those digits will result in a slightly longer execution, indicating the first correct digit
- Using this result, she can repeat the same test for the second (third, fourth) digit
- Complexity: We need a maximum of 40 tests (vs 9999 tests for an exhaustive search)
- The side-channel exploited by the attacker is the execution time ⇒ timing attack





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Power Consumption of a CMOS Circuit The inverter

Given input x = 0 V_{dd} $\rightarrow V_{\star} = 0$ → nMOS is blocking → pMOS is open $\rightarrow V_v = V_{dd}$ \rightarrow Logic output is y = 1X Given input x = 1 $\rightarrow V_x = V_{dd}$ → nMOS is open → pMOS is blocking V_{ss} $\rightarrow V_{v} = 0$ \rightarrow Logic output is y = 0





Power Consumption of a CMOS Circuit Energy dissipation



Rising edge

Falling edge



Power Consumption of a CMOS Circuit Information leakage

- Except for static leakage current, a CMOS circuit only consumes power during state changes of its gates (dynamic power consumption)
- By observing the power consumption of a circuit, we can deduce its activity
- Note that the number of gates changing their output depends on both the operations and the manipulated data
- Thus, the power consumption can reveal information on the executed operations and the involved data, including secrets



Hamming Distance

Definition

Given two bit vectors of equal length $A = \langle a_0 \ a_1 \dots a_{n-1} \rangle$ and $B = \langle b_0 \ b_1 \dots b_{n-1} \rangle$, their Hamming Distance is defined as the number of positions where they differ:

$$HD(A,B) = \sum_{i=0}^{n-1} a_i \oplus b_i$$

HD is a good approximate model for the power consumption of a register update in CMOS logic



Hamming Weight

Definition

Given a bit vector $A = \langle a_0 a_1 \dots a_{n-1} \rangle$, its Hamming Weight is defined as the number of bits with value one:

 $HW(A) = \sum_{i=0}^{n-1} a_i$

- *HW* is a good approximate model for the power consumption of a bus that is set to zero (or to high impedance) between transactions
- Note that HW(A) = HD(A, 0) and $HD(A, B) = HW(A \oplus B)$



Simple Power Analysis (SPA) Example: RSA

- Modular exponentiation algorithm 1: Inputs : M, K2: R = 1; 3: for i = |K| - 1; $i \ge 0$; i - -do4: $R = R^2$; 5: if $K_i == 1$ then 6: $R = R \times M$; 7: end if 8: end for 9: Return $R = M^K$;
 - Power consumption profile







- Recovery of the full secret (i.e. the key in case of RSA) with a single measurement
- Information is leaked due to different operations depending on the secret (multiply vs square) with a different power consumption profile.
- This type of attack using a single measure is called Simple Power Analysis
- Note that the computation time also leaks some information (difficult to exploit in this case)



Contents

Introduction

Power Consumption and EM Radiation Introduction Simple Power Analysis

Differential Power Analysis

Basic Principles The Leakage Model Analysis Algorithm

Further Attacks

Generalizations of DPA Computation Time

Conclusion



Differential Power Analysis

- Often, the leakage is not as obvious
- Need to use a large number of measures
- Need to use statistical tools
- This type of attack is called DPA (*Differential Power* Analysis)
- There are several variants (CPA, ...)



DPA: The Ingredients

Leakage Model *M* A model (function) predicting the behavior of the observed side-channel of the system, depending on a hypothesis on the system state

Distinguisher \mathcal{D} Statistical tool that allows to detect a correlation between the real system's behavior and our prediction

- Since the internal state of the system in particular the secret is unknown to the attacker, we need to make a hypothesis
- This hypothesis can be correct or wrong
- The distinguisher allows us to tell the good hypothesis (correct key) from the wrong ones (wrong keys)



DPA Manual 1/2

- 1. Determine a sensitive variable <u>S</u> depending on a part of the secret and on known inputs or outputs.
- 2. Establish a leakage model $\mathcal{M}(S)$ depending on S.
- 3. Perform observations (measurements) of the circuit's behavior on the considered side-channel, varying the known inputs or outputs.





DPA Manual 2/2

- 4. Analyze the data: For each possible value of S
 - For each known input/output *P* used during the observations, calculate *M*(*S*, *P*)
 - Use the distinguisher D to check if there is a correlation between the behavior predicted by the leakage model (depending on the hypothesis) and the real world observations
- For the correct value of <u>S</u>, the leakage model predicts correctly the circuit's behavior. As a consequence, the observations will be correlated to the model, and the distinguisher will detect this correlation.
- For all other (wrong) values of <u>S</u>, the model does not predict correctly the behavior, and there will be no correlation between the model and the observations.



DPA Overview



- I_i: Plain text message (or other known inputs/outputs)
- W_i: Measured power consumption (power trace)
- *M*: Leakage model, depending on secret *S* (and possibly known inputs/outputs)
- Find a correlation between and



Performing a DPA Attack

- 1. Which leakage model to choose?
- 2. Which distinguisher to choose?
- 3. How to perform the measurements?





- Context: Hardware implementation of DES (*Data Encryption Standard*) in ECB mode
- What we are looking for: key (56 bits)
- The adversary can send plain text messages to the circuit
- She can read the cipher text and measure the power consumption during the encryption
- Used attack: DPA (Differential Power Analysis)



Example: DPA vs DES DES: algorithmic view





DES: iterative hardware implementation



- **IP** Initial permutation
 - F Feistel function
- SK; Sub-key (round key)







- E Extension (32 to 48 bits)
- P Permutation (bit shuffling)
- Si Substitution





Power consumption model

- How to construct *M*?
- Power consumption during encryption operation
- Problems
 - DES is not alone on the chip (I/O...)
 - Power consumption of DES heavily depends on the key (56 bits), but we cannot test all 2⁵⁶ hypotheses (that's just brute force...)
- We need to concentrate on the power consumption of a part of the circuit, depending on a part of the key
- We consider the power consumption of the remaining circuit elements as noise





Value change of the state registers (L_i et R_i) during an encryption operation (first round)







Power consumption of the state registers



- Power consumption of register R_i at time T_1 : $P_{R_i}(T_1) = \delta \times HD(R_0, L_0 \oplus F(R_0, SK_0))$
- Known variables: R₀ et L₀ (depending directly on plain text)
- Unknown variables: SK_0 (48 bits of the key K), T_1 , and δ
- Still too many hypotheses: 2⁴⁸





Zoom on the Feistel function



How to construct a power consumption model depending on fewer bits of the secret key?





[12,27,1,17]









Power consumption of state registers (impact SBox 2)



- Considering bits [12,27,1,17] of register *R_i*
- Before T₁, their value depends on R₀ and thus directly on the (known) plain text
- After T_1 , their value depends on
 - Bits [12,27,1,17] of *L*₀ (known)
 - Bits [3,4,5,6,7,8] of R₀ (known)
 - Bits [6,7,8,9,10,11] of *SK*₀ (unknown)



Power consumption model HD on 4 bits

- Power consumption model: $P_{R_i[12,27,1,17]}(T_1) = \delta \times HD(R_0[12,27,1,17], L_0[12,27,1,17] \oplus F(R_0[3,4,5,6,7,8], SK_0[6,7,8,9,10,11])$
- Depends on a hypothesis on 6 bits of the first round key (2⁶ = 64 possible hypotheses)
- This model is only valid at instant T₁
- **5** possible output values (Hamming distance on 4 bits): $\{0, \delta, 2\delta, 3\delta, 4\delta\}$
- In the following, we suppose $\delta = 1$
- Finally: $P_4(I, S) = P_{R_i[12,27,1,17]}(T_1)$, where
 - I is the plain text
 - *S* is the hypothesis on *SK*₀ [6, 7, 8, 9, 10, 11]



Power consumption model vs actual power consumption

- Our model only predicts the power consumption of a small part of the circuit (4 flip flops) and only at one precise moment (*T*₁)
- Actual power consumption at T_1 :

 $P_{real}(I, \boldsymbol{K}, T_1) = P_4(I, S_{good}) + P_{rest}(I, \boldsymbol{K}, T_1),$

where S_{good} corresponds to the good hypothesis (correct value of $\frac{SK_0}{SK_0}$ [6, 7, 8, 9, 10, 11] depending on $\frac{K}{K}$)

■ We suppose that *P*_{rest}(I, *K*, *T*₁) is statistically independent of *P*₄(I, *S*_{good})





- For the good hypothesis on S (S_{good}), at instant T₁, the actual power consumption depends partially on our model P₄(I, S)
- This dependency is weak, so we need a lot of measurements in order to detect it using the distinguisher
- Perform N measurements (with constant key) for varying plain text messages I₁,..., I_N





- Power measurement during one encryption operation = power trace
- Trace = vector of samples: W(I_i, K, t) for t = 0, ..., T - 1 (with T the number of samples per trace)

 $W(I_i, K, t) = P_{real}(I_i, K, t) + Noise_{measure}$

In the following, we assume that the traces are aligned, i.e. that the index of the sample corresponding to instant *T*₁ is the same for all traces







Arbitrary units (x: time, y: power consumption)





- 1. Make a hypothesis on $S = S_H$ (64 possible values, including the good one: S_{good})
- Partition the set of traces depending on the prediction of the power consumption model: for each trace W(I_i, K, t) (i = 1,..., N)
 - Compute the power consumption model: P₄(I_i, S_H) (5 possible values)
 - Classify the trace in one of 5 sets $E_{P_4=0}, \ldots, E_{P_4=4}$:

$$E_{P_{4}=j} = \{W(\mathbb{I}_{i}, \mathbf{K}, t) \mid P_{4}(\mathbb{I}_{i}, S_{H}) = j\}$$





3. For each of the 5 sets, compute a mean trace (each sample *i* of the mean trace is the arithmetic mean of the *i*-th sample of all the traces in this set):

$$\overline{W}_{P_4=j}(t) = \frac{1}{n} \sum_{W \in E_{P_4=j}} W(\mathbb{I}_i, \mathbf{K}, t)$$

for t = 0, ..., T - 1 and with $n = |E_{P_4=j}|$ the number of traces in $E_{P_4=j}$





4. Compute a differential trace (for each hypothesis):

 $W_{\Delta}(t) = -2 \times \overline{W}_{P_{4}=0}(t) - \overline{W}_{P_{4}=1}(t) + \overline{W}_{P_{4}=3}(t) + 2 \times \overline{W}_{P_{4}=4}(t)$

for t = 0, ..., T - 1

- 5. Then find the maximum sample in the differential trace: $\mathcal{D}(S_H) = \max_t W_{\Delta}(t)$
- 6. Finally, we need to find out for which hypothesis on S, $\mathcal{D}(S_H)$ is maximal. This should be the good hypothesis: $S_{good} = \arg \max \mathcal{D}$



Example of a differential trace



64 differential traces superposed for SBox 2





We have:

 $W(I_i, K, t) = P_{real}(I_i, K, t) + Noise_{measure}$

• At time instant T_1 : $P_{real}(I, K, T_1) = P_4(I, S_{good}) + P_{rest}(I, K, T_1)$

It follows:

 $W(I_i, K, T_1) = P_4(I_i, S_{good}) + P_{rest}(I_i, K, T_1) + Noise_{measure}$

We consider the measurement noise and the power consumption of the rest of the circuit globally as noise:

$$W(I_i, K, T_1) = P_4(I_i, S_{good}) + Noise$$

Why does it work? (good hypothesis)

- Let's suppose we make the correct hypothesis on S (i.e. $S_H = S_{good}$)
- If we apply the power consumption model, it correctly predicts, for each observation, the behavior of 4 bits of the state register
- Therefore, the partitioning of the whole set of traces is consistent with the real behavior of these 4 bits:
 For *j* ∈ {0,...,4}, ∀W ∈ E_{P4=j}, we have:

 $W(I_i, K, T_1) = j +$ Noise











When we compute the mean traces, this consistency is preserved:

$$\overline{W}_{P_4=j}(T_1)=j+$$
 Noise

The equation of the differential trace distinguishes this coherence for the sample correspoding to T₁:

$$W_{\Delta}(T_1) = -2 \times \overline{W}_{P_4=0}(T_1) - \overline{W}_{P_4=1}(T_1) + \overline{W}_{P_4=3}(T_1) + 2 \times \overline{W}_{P_4=4}(T_1)$$
$$= -2 \times (0 + \boxed{\text{Noise}}) - (1 + \boxed{\text{Noise}}) + (3 + \boxed{\text{Noise}}) + 2 \times (4 + \boxed{\text{Noise}})$$
$$\approx 10$$





Why does it work? (good hypothesis)

 $\overline{W}_{P_{4}=0}(T_{1}) \overline{W}_{P_{4}=1}(T_{1}) \overline{W}_{P_{4}=2}(T_{1}) \overline{W}_{P_{4}=3}(T_{1}) \overline{W}_{P_{4}=4}(T_{1})$



 $W_{\Delta}(t) = -2 \times \overline{W}_{P_{\mathbf{4}}=0}(t) - \overline{W}_{P_{\mathbf{4}}=1}(t) + \overline{W}_{P_{\mathbf{4}}=3}(t) + 2 \times \overline{W}_{P_{\mathbf{4}}=4}(t)$



Why does it work? (bad hypothesis)

- Now suppose we have made a wrong hypothesis on $S(S_H \neq S_{good})$
- When applying the power consumption model, it does not predict correctly the power consumption of the state register
- Therefore, the partitioning of the traces is inconsistent with the real behavior of the state register:
 For *j* ∈ {0,...,4}, ∀W(I_i, K, t) ∈ E_{P4=j}, we have:

$$W(I_i, K, T_1) = k_i +$$
Noise

for some $k_i \in \{0, ..., 4\}$





power $(W_i(T_1))$



Why does it work? (bad hypothesis)

As a consequence of the inconsistent (more or less random) partitioning, the mean traces of the different partitions are identical:

$$\overline{W}_{P_4=j}(T_1) = 2 +$$
Noise

The equation for the differential trace results in a value around 0:

$$W_{\Delta}(T_1) = -2 \times \overline{W}_{P_{\mathbf{4}}=0}(T_1) - \overline{W}_{P_{\mathbf{4}}=1}(T_1) + \overline{W}_{P_{\mathbf{4}}=3}(T_1) + 2 \times \overline{W}_{P_{\mathbf{4}}=4}(T_1)$$
$$= -2 \times (2 + \boxed{Noise}) - (2 + \boxed{Noise}) + (2 + \boxed{Noise}) + 2 \times (2 + \boxed{Noise})$$
$$\approx 0$$

This is also the case for all other samples which do not correspond to T₁, for good and bad hypotheses





$\overline{W}_{P_{4}=0}(T_{1}) \overline{W}_{P_{4}=1}(T_{1}) \overline{W}_{P_{4}=2}(T_{1}) \overline{W}_{P_{4}=3}(T_{1}) \overline{W}_{P_{4}=4}(T_{1})$





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COMELEC / SSH



As a conclusion, all samples of all differential traces are approximately zero except for the one corresponding to time instant T₁ for the good hypothesis on S





DPA in a Nutshell

- 1: **Inputs**: Model \mathcal{M} , traces W_i , inputs I_i for $1 \le i \le N$
- 2: for each hypothesis S_H on secret **S** do
- 3: for $i \in \{1, ..., N\}$ do
- 4: $j \leftarrow \mathcal{M}(I_i, S_H)$
- 5: $E_{\mathcal{M}=j} \leftarrow E_{\mathcal{M}=j} \cup \{W_i\}$
- 6: end for
- 7: for $j \in \operatorname{range} \mathcal{M}$ do
- 8: compute mean trace $\overline{W}_{\mathcal{M}=j}$
- 9: end for
- 10: compute differential trace W_{Δ}
- 11: $\mathcal{D}(S_H) \leftarrow \max_t W_{\Delta}(t)$
- 12: end for
- 13: $S_{good} \leftarrow \arg \max \mathcal{D}$
- 14: Return Sgood





- We have recovered 6 bits of SK₀, which gives us directly 6 bits of K
- By repeating the attack on the other S-boxes, we can recover all 48 bits of SK₀, and therefore 48 bits of K
- For the remaining 8 bits, we can attack the second round (the first round is now entirely known), or just do an exhaustive search
- Total complexity of the attack: 64 hypotheses for each of the 8 S-boxes plus exhaustive search: 64 × 8 + 256 operations¹



¹What is the complexity of one operation?

Contents

Introduction

Power Consumption and EM Radiation

Simple Power Analysis

Differential Power Analysis

Basic Principles The Leakage Model Analysis Algorithm

Further Attacks

Generalizations of DPA Computation Time

Conclusion



Leakage Models

- **Hamming weight:** $\mathcal{M}(S) = HW(S)$
 - Suitable for buses which are reset to zero (or high impedance) after transmission
- Hamming distance [2]:
 - $\mathcal{M}(S) = \mathrm{HD}(S, S_{-1}) = \mathrm{HW}(S \oplus S_{-1})$
 - Suitable for hardware implementations (CMOS power consumption)
- Switching distance [7]: $\mathcal{M}(S) = 1$ for transition $0 \to 1$, and (1δ) for transition $1 \to 0$, else 0
 - Suitable for near field EM





Partitioning

- Difference of means [6]: DPA
- Covariance [1]
- Mutual information [4]: MIA
- Comparison
 - Correlation [2]: CPA



Correlation Power Analysis (CPA)

PEARSON correlation coefficient

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y},$$

where cov(X, Y) = E[(X - E[X])(Y - E[Y])].

If there is a linear dependence between the prediction of the leakage model and the real behavior of the circuit, the linear correlation coefficient can be used to test the hypothesis



Correlation Power Analysis (CPA)



Good key hypothesis \Rightarrow correlation \neq 0



Correlation Power Analysis (CPA)



Bad key hypothesis \Rightarrow correlation \approx 0



Timing Attacks

- Attacks based on power consumption or EM radiation require physical access to the target device
- In contrast, timing attacks can be performed remotely, including over a network
- Examples:
 - · Remote key recovery over the network [3]
 - Key recovery from another virtual machine running on the same host [5]
- Possible sources of timing variations:
 - Algorithmic
 - Hardware optimizations of the host processor: cache, pipeline, ...



Timing Attacks

Example: Attacking RSA over the network [3]

- RSA in OpenSSL (version 0.9.7)
- Due to some optimizations (Chinese remainder theorem, Montgomery reduction, sliding window expoentiation, Karatsuba multiplication) the execution time slightly depends on the secret key
- The attack has been demonstrated locally and remotely over a network
- Taking the mean of many tries, the latency and jitter introduced by the network are not sufficient to mask the small timing variations
- More attacks in the μ -architecture chapter



Contents

Introduction

Power Consumption and EM Radiation

Introduction Simple Power Analysis

Differential Power Analysis

Basic Principles The Leakage Model Analysis Algorithm

Further Attacks

Generalizations of DPA Computation Time

Conclusion



Conclusion

- Physical implementations leak information on various side-channels
 - Power
 - EM radiation
 - Timing
 - . . .
- If the leakage depends on sensitive data (such as a cryptographic key), it can be exploited by a side-channel attack
- These attack mostly require physical access to the target system
- Statistical side-channel attacks can be very effective



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