



# Real-Time Scheduling for Multi-Processors Systems

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Version 1.0

# Architecture Issues

## No Mono-Processor Architecture Anymore

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- Historically ... mono-processors
  - platform = a dedicated processor, a clock and a common memory ...
  - predictable but expensive and technologically limited power.
- Trends ... multi-processors
  - More powerful, but less predictable (cache, pipeline)
  - Use COTS (not dedicated) processors (FAA, 2011).
  - Several identical processors (or not).
  - Shar resources: cache, memory, bus, crossbar, network-on-chip => +interferences ; -predictable



# Architecture Issues

## Interferences with Multi-Processors

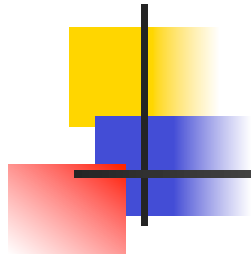
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- Let's have task  $T_1$  (resp  $T_2$ ) running on core  $C_1$  (resp  $C_2$ ) ;  $C_1$  and  $C_2$  share a common cache  $L_2$  or an interconnection bus
- $T_1$  and  $T_2$  are functionally independent ... but finally dependent because of shared hardware resources inducing interferences
- A task can be delayed due to contention / interference on shared hardware
- This can be an even more important problem in multi-processors than in mono-processor

# Multi-Processors Architecture

## Processors

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- Identical processors: processors all executing the same units of work during the same units of time
- Uniform processors: processor  $j$  with speed  $s_j$  executes  $s_j \cdot t$  units of work for  $t$  units of time.
- Heterogeneous processors: processor  $j$  executes  $s_{i,j} \cdot t$  units of work of job  $i$  for  $t$  units of time.
- Heterogenous processors : no shared memory, nor migration (a distributed system)

# Multi-Processors Architecture Scheduling



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- Mono-Processor scheduling : 1 problem
  - Time Allocation – when to execute a task
- Multi-Processors scheduling : 2 problems
  - Processor Allocation – where to execute a task
  - Time Allocation – when to execute a task

As a consequence, most results from  
mono-processor real-time scheduling theory  
are **no longer true** for  
multi-processors real-time scheduling theory



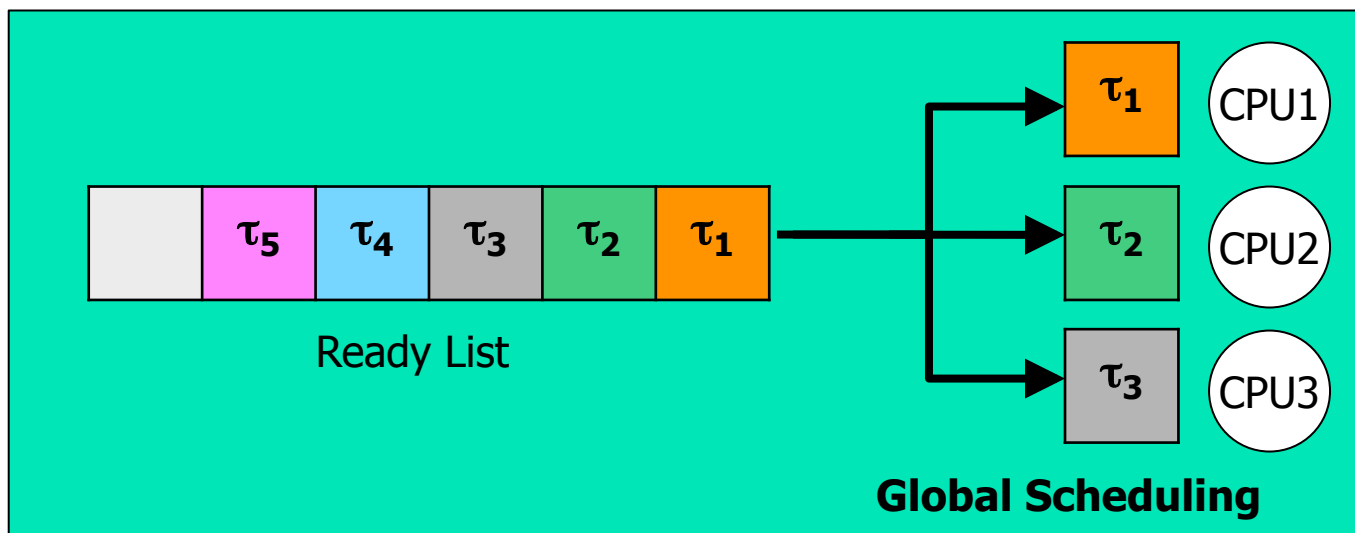
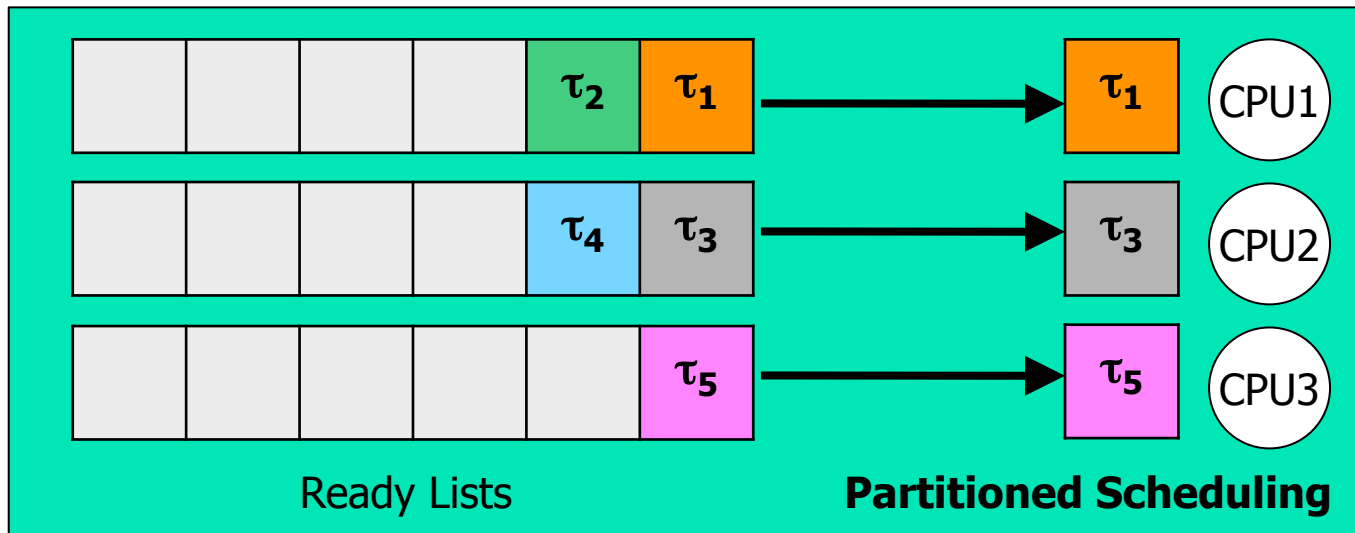
# Multi-Processors Scheduling Different Approaches

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- Partitioned scheduling (offline approach)
  - Handle separately processor and time allocations
    - Map all tasks on processors
    - Schedule tasks on each processor.
  - Possible end-to-end delay verification
- Global scheduling (online approach)
  - Handle globally processor and time allocations
  - Pick a task from a global ready list
  - Map it on one of the idle processors
- Hybrid scheduling (mixed approach)
  - Predefine a tree of Virtual Processes (servers)
  - Schedule tasks and the tree of Virtual Processes

# Scheduling Approaches

## Partitioned Scheduling Approach





# Partitioned Scheduling Task Assignment

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- How to statically assign tasks to processors
- Bin-packing problem: minimize the number of bags to pack bins of different volumes
- NP-hard problem => partitioning heuristics
  - Different parameters:
    - Processors (identical or not), tasks (periods, budgets), etc.
    - Task communications, shared resources, etc.
  - Different objective function:
    - Minimize processors, communications, latencies, etc.
- Difficult to compare heuristics
  - especially when the final objective is actually schedulability





# Partitioned Scheduling Assignment and Scheduling Variants

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- Sort tasks before packing
  - Ascending/descending order of utilization/period
- Select a mono-processor scheduling
  - RM or DM, EDF or LLF
  - Schedulability test to allocate a task to a processor
- Select a bin-packing heuristic
  - First-Fit, Next-Fit, Worst-Fit or Best-Fit



# Partitioned Scheduling

## Rate-Monotonic Next-Fit

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- List tasks in ascending order of their utilization/period.
- Processor  $p=0$
- For task  $t=0$  to  $n$ 
  - Assign task  $t$  to processor  $p$  if the feasibility test is met (eg:  $U \leq 0.69$  or response time computation)
  - Stop when no processor found
  - Loop to next processor  $p = (p+1) \bmod m$

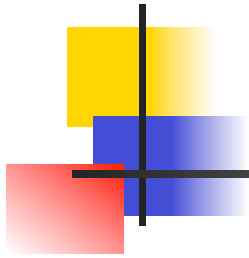
# Partitioned Scheduling Limitations

- Partitioned Scheduling cannot be optimal
- $m$  processors
- $(m+1)$  tasks of parameters  $(C, T)$ ,  $C=T/2+\varepsilon$
- Exercice : Prove that for periodic tasksets with implicit deadlines, the largest worst-case utilization bound for any partitioning algorithm is  $(m+1)/2$ .

# Partitioned Scheduling

## Pros and Cons

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- Pros
  - Better suitability for heterogeneous systems
  - Inherit from mature mono-processor scheduling
  - Time and space isolation (major safety property)
    - Failures / anomalies limited to one processor
- Cons
  - 2 problems both being NP-hard
    - Processor allocation (mapping)
    - Time allocation (scheduling)
  - Less optimal use of resources (idle processors)



# Partitioned Scheduling

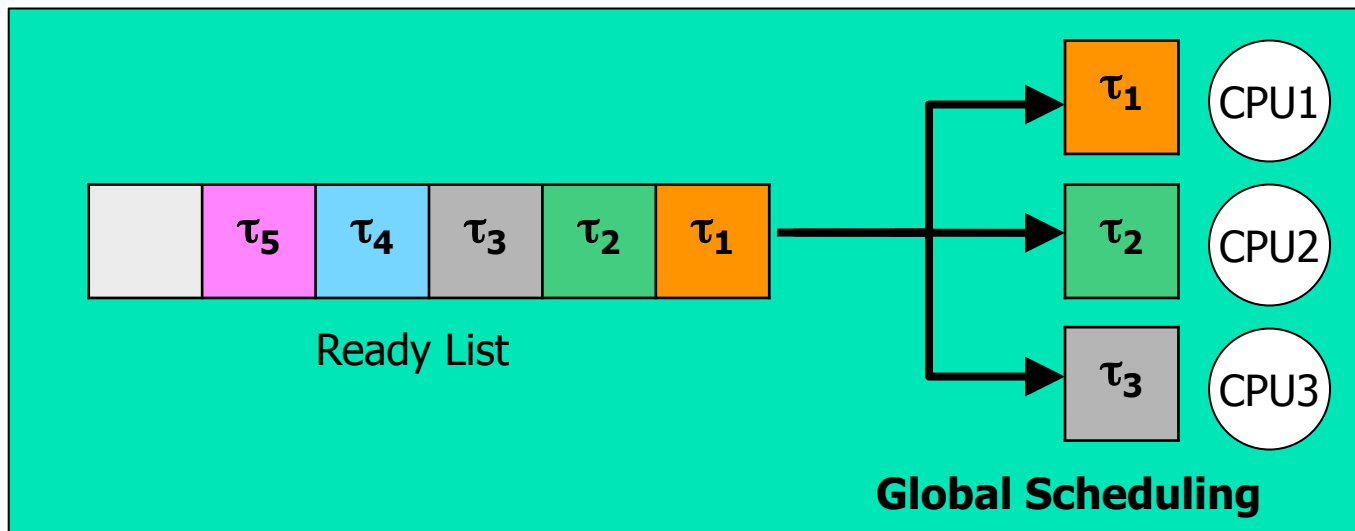
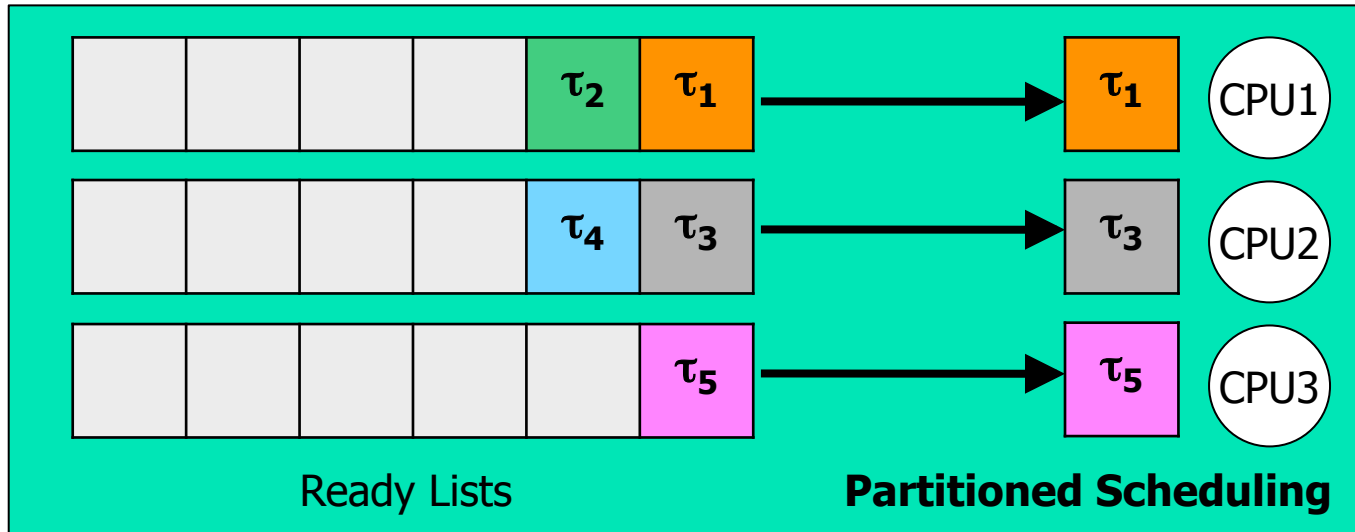
## Other resources (memory, bus, ...)

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- Similar benefits/limitations for other resources
  - resource partitioning and
  - resource sharing
- Resource partitioning : great predictability ... but resources less efficiently used
- Global resource sharing: poor predictability ... but resources more efficiently used
- Example: partitioned cache vs shared cache
  - Partition too small: time to reload data
  - Partition too large: waste of resource

# Scheduling Approaches

## Global Scheduling



# Global Scheduling Pros and Cons



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- Pros

- Optimal scheduling exist
- Better suited for homogeneous multi-core architectures
- Better resource optimization : busy cores, less preemptions ... but migrations

- Cons

- Not well suited at all to heterogeneous systems
- More recent and less numerous results of scheduling theory
- ... for simple architectures and task models



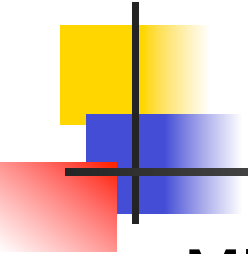
# Global scheduling

## Sharing resources

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- A global scheduler deals with two problems:
  - When and how to assign task / job priorities.
  - Choose a processor on which to run the task.
- Sharing time
  - Preemption (same as mono-processor)
  - A job starts its execution in a time interval and ends in another time interval
- Sharing processors
  - Migration
  - A job starts its execution on a processor and ends on another processor





# Global Scheduling Migrations and Priorities

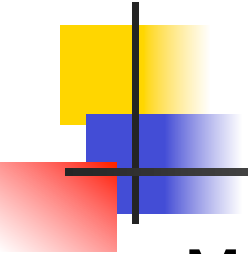
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## Migration strategies

- No task migration: All its jobs are assigned to a given processor => partitioning
- Task migration: Jobs can start executing on different processors but complete on their selected processor
- Job migration: A job can migrate during its execution.

## Priority assignments

- Fixed priority associated to a task (eg: RM).
- Fixed priority associated to a job (eg: EDF).
- Dynamic priority associated to a job (ex: LLF).



# Global Scheduling

## Two general approaches

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Mono-processor based global scheduling:

- Global RM, Global DM, Global EDF, Global LLF, ...
  - Variants depending on migration level (task or job)
- Globally apply a mono-processor scheduling strategy on all processors. Assign the  $m$  highest priority tasks or jobs to the  $m$  processors at any time.
- Task or job preemption when all processors are busy

New algorithms: PFair, RUN, ...

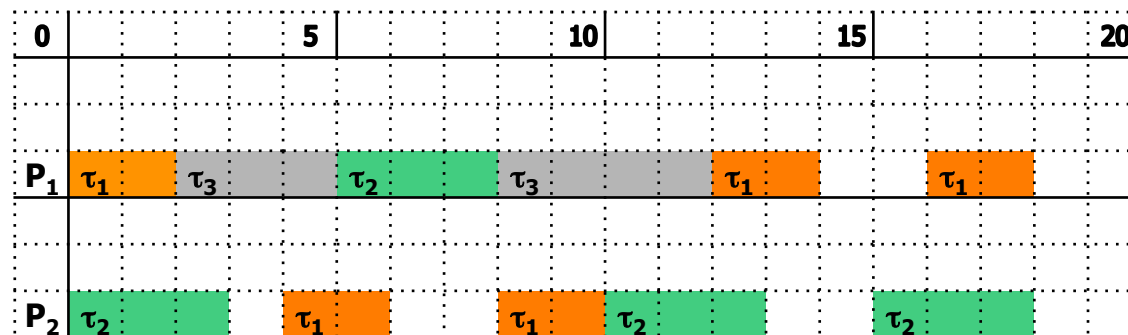
**Different and fewer results and properties  
compared to mono-processor scheduling**

# Mono-Processor based Global Scheduling

## Different response times

- Use of Global Deadline Monotonic scheduling
- Priority assignment:  $\tau_1 > \tau_2 > \tau_3$
- Task can migrate, jobs cannot

	C	T	D
$\tau_1$	2	4	4
$\tau_2$	3	5	5
$\tau_3$	7	20	20

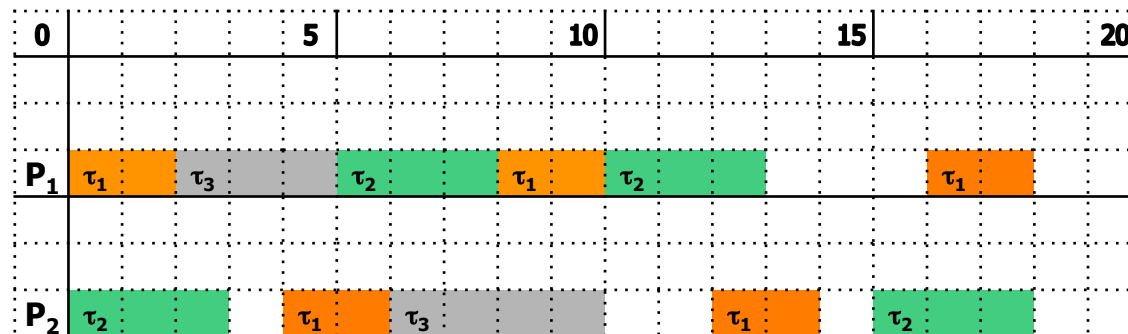


# Mono-Processor based Global Scheduling

## Different response times

- Use of Global Deadline Monotonic scheduling
- Priority assignment:  $\tau_1 > \tau_2 > \tau_3$
- Jobs can migrate
- Not the same response time for  $\tau_3$

	C	T	D
$\tau_1$	2	4	4
$\tau_2$	3	5	5
$\tau_3$	7	20	20

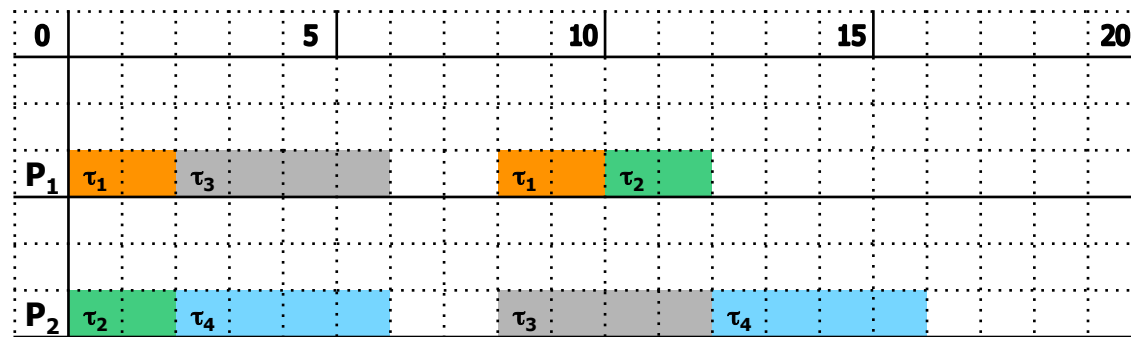


# Mono-Processor based Global Scheduling

## No Critical Instant

- In a mono-processor, the critical instant is the worst case scenario for periodic tasks
- All tasks are released at the same instant
- Used to compute the worst response time
- But not the worst scenario in multi-processors.
- Here,  $R_4=8$  but with critical instant  $R_4=6$

	C	D	T
$\tau_1$	2	2	8
$\tau_2$	2	4	10
$\tau_3$	4	6	8
$\tau_4$	4	7	8





# Mono-Processor based Global Scheduling

## Different feasibility interval

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- In mono-processor, the feasibility interval is used to check schedulability of independent asynchronous / synchronous periodic tasks,  $\forall i: D_i \leq P_i$  with a fixed priority scheduling  $[0, 2 * \text{LCM} (\forall i: P_i) + \max (\forall i: S_i)]$
- In multi-processors, a similar result:  $[0, \text{LCM} (\forall i: P_i)]$  but for a set of independent synchronous periodic tasks only



# Mono-Processor based Global Scheduling

## Scheduling anomalies

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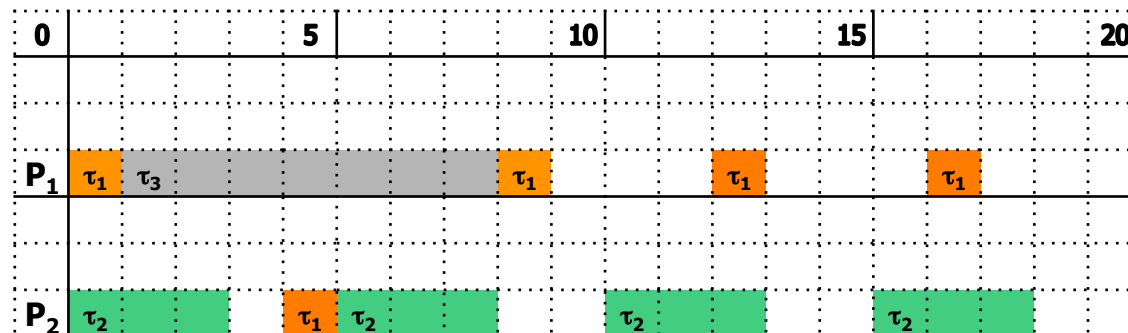
- Anomaly: intuitively positive change in a schedulable set of tasks that leads to a non-schedulable set of tasks
- In mono-processor, when a tasks set is schedulable, it is still schedulable if we lower its utilisation (reduce  $C_i$  or increase  $T_i$ )
- In multi-processor, this is no longer true

# Mono-Processor based Global Scheduling

## Scheduling anomalies

- Use of Global Deadline Monotonic Scheduling
- Jobs can migrate
- $U_1 = 1/4$
- Tasks set is schedulable

	C	T	D
$\tau_1$	1	4	2
$\tau_2$	3	5	3
$\tau_3$	7	20	8



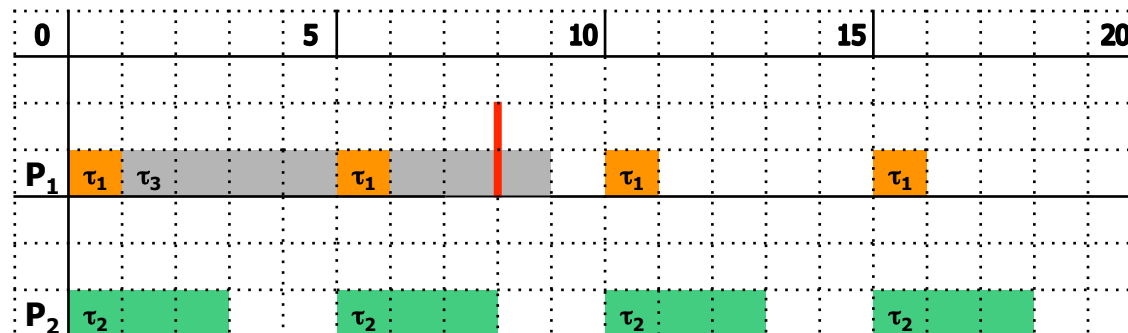


# Mono-Processor based Global Scheduling

## Scheduling anomalies

- Use of Global Deadline Monotonic Scheduling
- Task  $\tau_1$  has a larger period
- Task set with a lower utilisation ( $1/4 \rightarrow 1/5$ )
- Tasks set is non-schedulable ( $R_3=9 > D_3$ )

	C	T	D
$\tau_1$	1	5	2
$\tau_2$	3	5	3
$\tau_3$	7	20	8



# Mono-Processor based Global Scheduling Limitations



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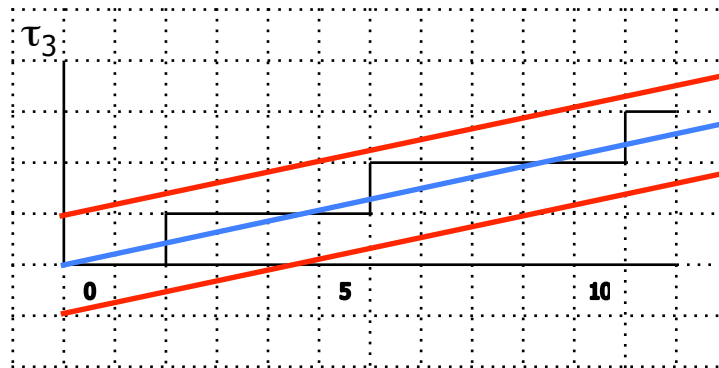
- $m$  processors
- $(m+1)$  tasks of parameters  $(C, T)$ ,  $C = T/2 + \epsilon$
- Exercice : Prove that the maximum utilization bound for any global fixed job priority algorithm is  $(m+1)/2$ .

# Global Scheduling

## Pfair Algorithms : Principles

- The proportion of time units allocated at instant  $t$  to a task must remain as close as possible to its utilisation
- Optimal algorithm for identical processors and synchronous deadline implicit periodic tasks
- Lots of context switches and migrations

	C	T
$\tau_1$	1	2
$\tau_2$	1	3
$\tau_3$	2	9





# Global Scheduling

## Pfair Algorithms : Modelling

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- Execute tasks at a constant rate (fluid model) such as  $\forall i: \text{workload}(\tau_i, t) = t * C_i / T_i$
- Can be approximated by  $\text{sched}(\tau_i, t)$  where  $\text{sched}(\tau_i, t) = 1$  when  $\tau_i$  is scheduled in interval  $[t, t + 1[$ ,  $\text{sched}(\tau_i, t) = 0$  otherwise
- A schedule is said to be Pfair if and only if  $\text{lag}(\tau_i, t) = \text{workload}(\tau_i, t) - \sum_{k \leq t} \text{sched}(\tau_i, k)$  where  $\forall i, \forall t: -1 \leq \text{lag}(\tau_i, t) \leq 1$
- A Pfair scheduling is feasible on  $m$  processors as long as  $U \leq m$  (full utilization !)



# Global Scheduling

## Pfair Algorithms : Implementation

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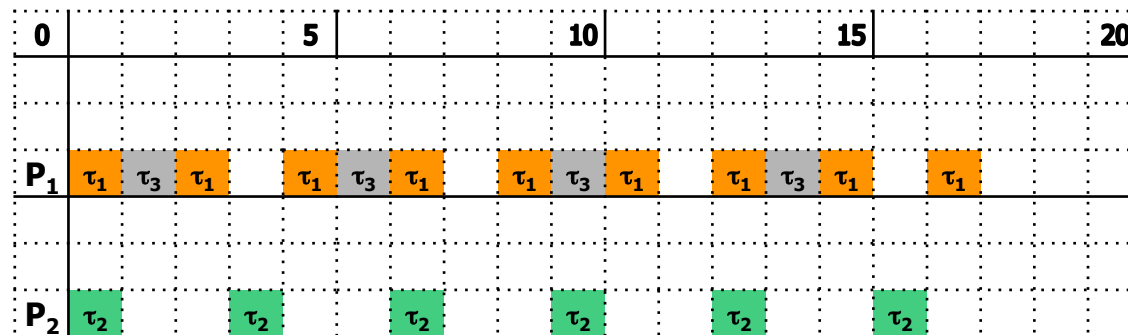
- Split each task  $i$  into  $C_i$  subtasks (1 time unit)
- Assign a pseudo deadline  $d(\tau_i, j)$  and a pseudo release  $r(\tau_i, j)$  to subtask  $j$  in  $[1..C_i]$ :
  - $d(\tau_i, j) = \lceil j * T_i / C_i \rceil$
  - $r(\tau_i, j) = \lfloor (j - 1) * T_i / C_i \rfloor$
- Schedule subtask  $j$  according to  $d(\tau_i, j)$  (EDF)
- Improve Pfair with non-arbitrary tie breaks to reduce context switches and migrations in case of identical pseudo-deadlines

# Global Scheduling

## Pfair Algorithms : Example

- $r(\tau_i, j) = \lfloor (j-1) * T_i/C_i \rfloor$  and  $d(\tau_i, j) = \lceil j * T_i/C_i \rceil$
- $r(\tau_1, 1) = 0$  ;  $d(\tau_1, 1) = 2$  ;  $U_1=1/2$
- $r(\tau_2, 1) = 0$  ;  $d(\tau_2, 1) = 3$  ;  $U_2=1/3$
- $r(\tau_3, 1) = 0$  ;  $d(\tau_3, 1) = 5$  ;  $U_3=2/9$
- $r(\tau_3, 2) = 4$  ;  $d(\tau_3, 2) = 9$  ;  $U_3=2/9$

	C	T
$\tau_1$	1	2
$\tau_2$	1	3
$\tau_3$	2	9



# Global Scheduling Conclusions

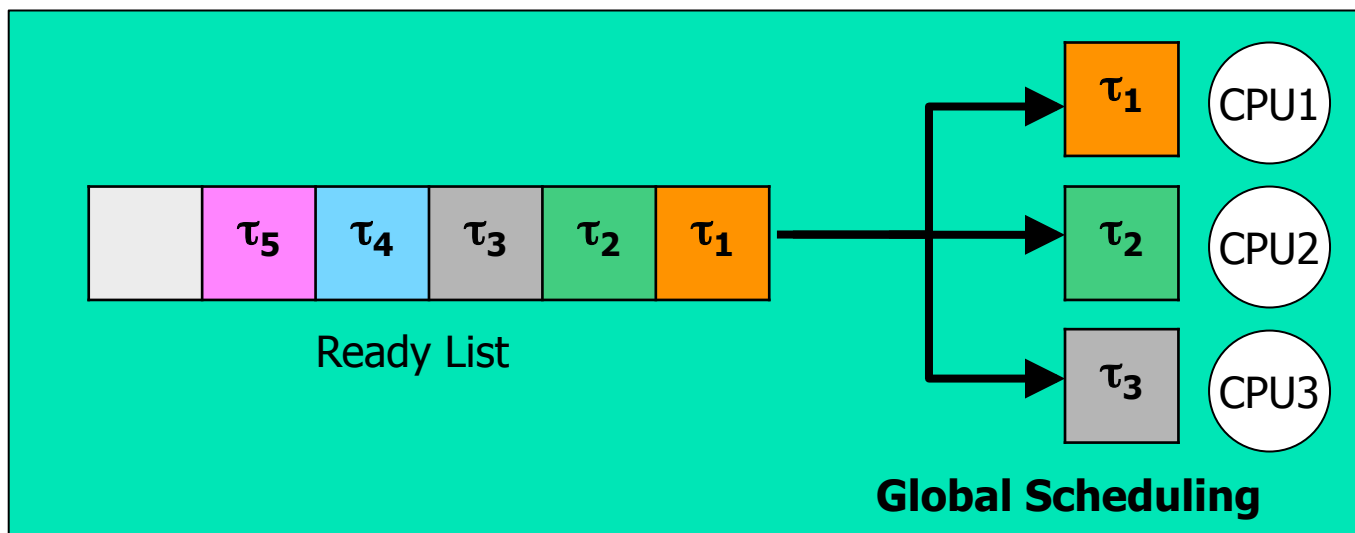
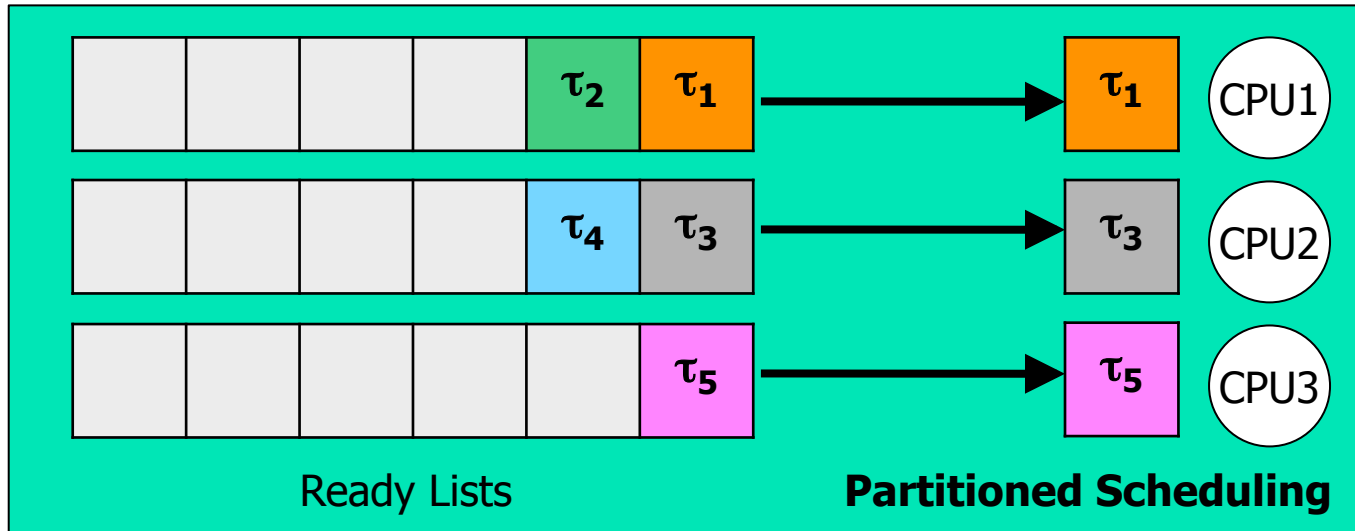


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- Global multi-processor scheduling has different properties compared to mono-processor scheduling (optimality, critical instant, feasibility interval, anomalies, ...).
- Additional parameters : migration, task / processor assignment, ...
- We limited architecture to identical processors, without shared resources
- We have limited task model to a simplified task one
- We have not discussed dependencies between tasks (shared resources, precedence constraints), nor communications.

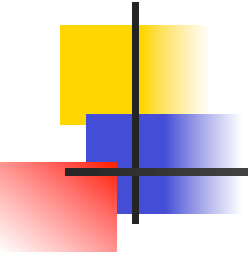
# Scheduling Approaches

## Hybrid Scheduling





# Hybrid Scheduling Principles

- 
- A mixed solution between partitioned (offline) and global scheduling (online)
  - Example: RUN (Reduction to Uniprocessor)
  - Optimal, less preemptions compared to PFair
    - Offline: build a reduction tree (PACK & DUAL steps)
      - Partition tasks on a min nbr of virtual processors (PACK)
      - Regroup the processors idle time as tasks (DUAL)
      - Pack new tasks to build idle virtual processors (PACK)
      - Stop when schedule on a single processor
    - Online: schedule on each (virtual) processor with EDF by reading the reduction tree.

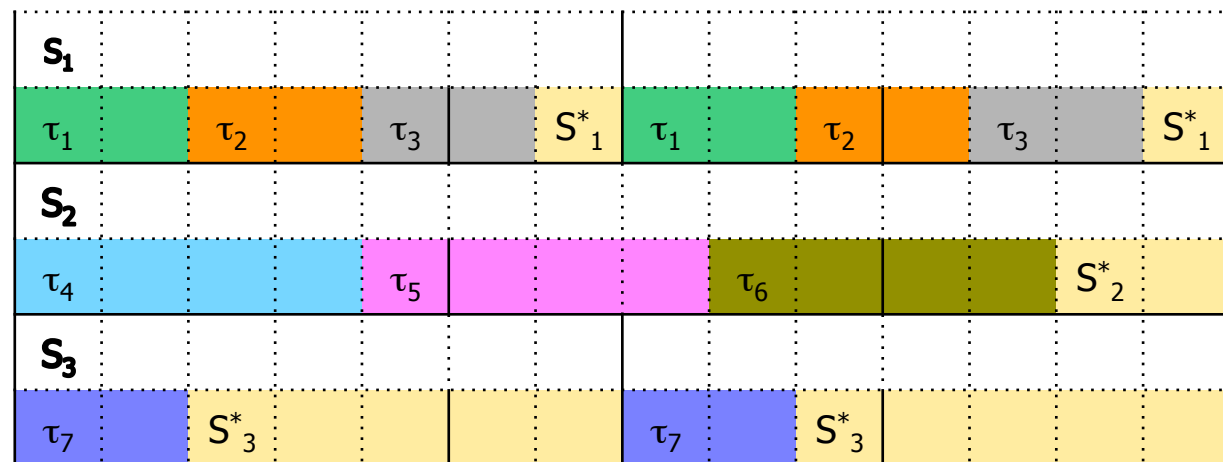
# RUN

## Offline : PACK (first and initial layer)

- Pack tasks on a minimum number of virtual processors (servers)  $S_1$  to  $S_3$ . Use First-Fit.
- So, we cannot merge 2 virtual processors (VP)
- 3 idle time intervals :  $S_1^*$  to  $S_3^*$

U=2

	C	T
$\tau_1$	2	7
$\tau_2$	2	7
$\tau_3$	2	7
$\tau_4$	4	14
$\tau_5$	4	14
$\tau_6$	4	14
$\tau_7$	2	7

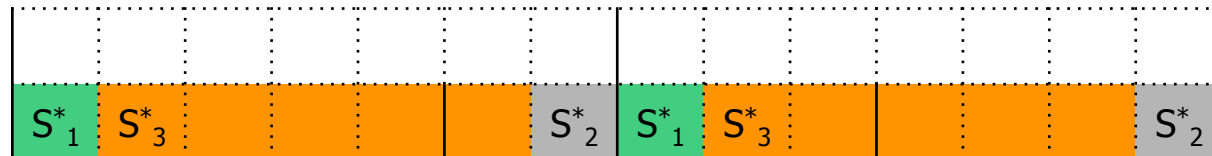


# RUN

## Offline : DUAL + PACK (second layer)

- Define  $S_1^*$  to  $S_3^*$  as (dual) tasks
- They model the idle time left on VPs
- Pack and schedule  $S_1^*$  to  $S_3^*$  on 1 VP
  - This new VP schedules « idle tasks » : we free a processor as the idle time is packed on 1 processor
- While #processors > 1, loop DUAL+PACK steps

	C	T
$S_1^*$	1	7
$S_2^*$	2	14
$S_3^*$	5	7



# RUN

## Online: Schedule Reduction Tree

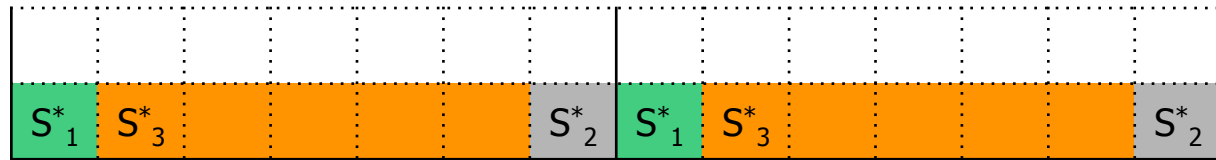
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- We have a tree of servers (or a hierarchy of servers) that schedules tasks and servers
  - We start scheduling the root server in the tree
  - When we schedule a dual server, we do not schedule its tasks or servers. We schedule the remaining tasks or servers according EDF.
  - When we schedule a primary server, we do schedule its tasks or servers according EDF
- In the example, we start executing  $S_1^*$ . Thus, we do not execute  $S_1$  but  $S_2$  ou  $S_3$ . According EDF,  $S_2$  will execute  $\tau_1$  and  $S_3$  will execute  $\tau_1$

# RUN

## Online: Scheduling a dual server

	C	T
$S_1^*$	1	7
$S_2^*$	2	14
$S_3^*$	5	7

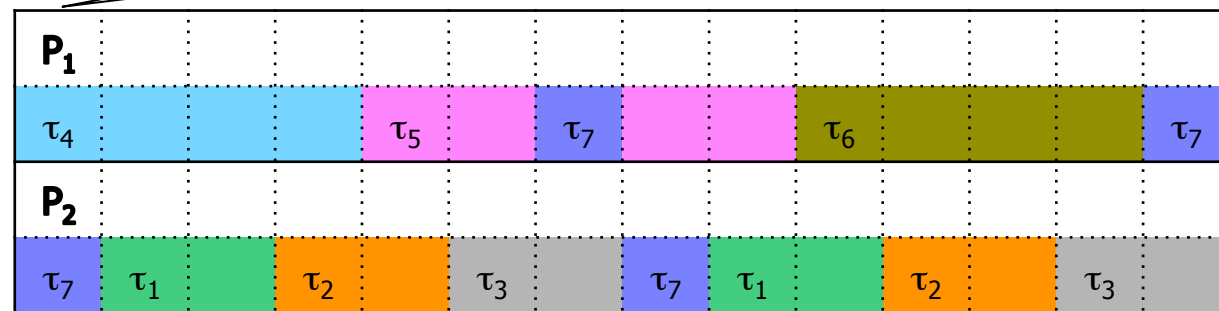


Schedule  $S_1^* \Rightarrow$  schedule all but  $S_1$ ,  
 $\Rightarrow$  schedule  $S_2$  or  $S_3$

Schedule  $T_4, T_5$  or  $T_6$  on  $P_1$  and  
 $T_7$  on  $P_2$  both using EDF

U=2

	C	T
$\tau_1$	2	7
$\tau_2$	2	7
$\tau_3$	2	7
$\tau_4$	4	14
$\tau_5$	4	14
$\tau_6$	4	14
$\tau_7$	2	7



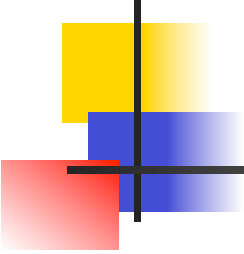
Laurent Pautet

# Real-Time Scheduling for Distributed Systems

- Tasks exchange messages
  1. Tasks are dependant and assigned to procs
    - The task input is the output of its predecessors
  2. Model and schedule messages as tasks

<b>Non-preemptive task</b>	<b>Message</b>
(Mono) Processor	Communication medium
Capacity / Budget	Communication delay (buffer, access, propagation)

3. Schedule messages on bus or network
  - Use non-preemptive tasks scheduling

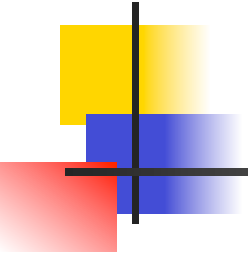


# Distributed Real-Time Scheduling

## Step 1: Dependant Tasks on Mono-Processors

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- Dependant tasks on a mono-processor
  - Modify task parameters to have independant tasks
  - $A_i^* = \max(A_i, \max_{j \text{ in pred}(i)} A_j^* + C_j)$
  - $D_i^* = \min(D_i, \min_{j \text{ in succ}(i)} D_j^* - C_j)$
- For a static priority scheduling, give higher priority to predecessors than to task (DMS)
  - We can compute response time
- For a dynamic priority scheduling, use new deadlines (EDF)



# Distributed Real-Time Scheduling

## Step 2 : Dependant tasks on Distributed Systems

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- Holistic Method
  - Compute response time with jitter ...
  - defined as the max response time of predecessors
- Iterative method (as for mono-processors)
- For task
  - $R_i^{n+1} = J_i + C_i + \sum_{k \text{ in pred}(i)} C_k * [(J_k + R_i^n) / T_k]$
- For message
  - $R_i = J_i + M_i$

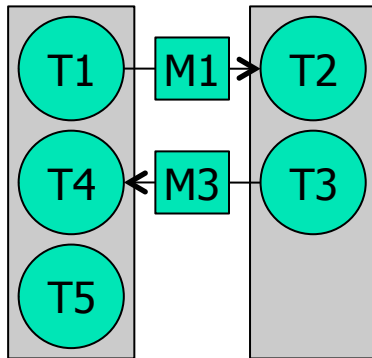


# Distributed Real-Time Scheduling

## Step 3 : Message Scheduling on (CAN) Bus

- Messages modeled as non-preemptive tasks
- Compute response time for static priority scheduling of non-preemptive tasks
- $$R_{n+1}^i = J_i + C_i + \sum_{k \text{ in } hp(i)} C_k * \lceil (J_k + R_n^i) / T_k \rceil + \max_{l \text{ in } lp(i)} (C_l)$$
  - The last term represents the blocking time induced by a lower priority non-preemptive task

# Distributed Real-Time Systems



Step 1 :  $T_1$  (resp  $T_3$ ) has higher priority than successor  $T_2$  (resp  $T_4$ )

$$\text{Step 2 : } R_i^{n+1} = J_i + C_i + \sum_{k \text{ in pred}(i)} C_k * [(J_k + R_i^n) / T_k]$$

	M1	M3	T1	T2	T3	T4	T5
J	0	0	0	0	0	0	0
R	6	1	4	5	2	9	12

	T	C	Pri
T1	100	4	HI
T2	100	3	ME
T3	60	2	HI
T4	60	5	ME
T5	90	3	LO
M1	100	6	LO
M3	60	1	HI

	M1	M3	T1	T2	T3	T4	T5
J	4	2	0	6	0	1	0
R	10	3	4	11	2	10	12

	M1	M3	T1	T2	T3	T4	T5
J	4	2	0	10	0	3	0
R	10	3	4	15	2	12	12

Step 3 :  $M_1$  and  $M_3$  are schedulable on network (ytivial)



# Conclusions

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- Less mono-processors, more multi-processors or heterogeneous systems on the market
- Very active research domain to design new scheduling approaches
- Less predictive processors on the market ; approximate WCET due to many interferences
- Define modes and change mode when overloaded
- The low criticality mode includes all the tasks
- The high criticality one only high criticality tasks
- Active research domain : mixed criticality Systems

